

Aharonov-Casher effect in quantum ring ensembles

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We study the transport of electrons through a single-mode quantum ring with electric field induced Rashba spin-orbit interaction that is subjected to an in-plane magnetic field and weakly coupled to electron reservoirs. Modelling a ring array by ensemble averaging over a Gaussian distribution of energy level positions, we predict slow conductance oscillations as a function of the Rashba interaction and electron density due to spin-orbit interaction-induced beating of the spacings between the levels crossed by the Fermi energy. Our results agree with experiments by Nitta *c.s.*, thereby providing an interpretation that differs from the ordinary Aharonov-Casher effect in a single ring.

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The Aharonov-Casher (AC) effect¹ is an analog of the Aharonov-Bohm (AB) effect, but caused by the spin-orbit interaction (SOI) rather than an external magnetic field. Originally, Aharonov and Casher predicted in 1984 that a spin accumulates a phase when the electric charge is circling in an external electric field.¹ This situation is similar to a single-mode ballistic ring with the Rashba spin-orbit interaction. Quantum rings in high-mobility semiconductor material have therefore attracted extensive attention, both experimentally and theoretically, as model devices to investigate fundamental quantum mechanical phenomena.

In the ordinary AC effect, the electrons injected into a quantum ring with SOI acquire spin phases when traversing the two arms due to precession in the effective spin-orbit magnetic field. Interference of the spinor wave functions at the exit point of the ring then leads to an oscillatory conductance as a function of the spin-orbit coupling constant that in Rashba systems can be tuned by an external gate voltage. The theory of AC conductance oscillations² in a single-mode quantum ring symmetrically coupled to two leads is in a good agreement with experimental observations.³ More recently, the zero magnetic field conductance behavior as a function of gate field has been interpreted in terms of the modulation of (electron density-independent) Altshuler-Aronov-Spivak (AAS) oscillations by the SOI,⁴ emphasizing the importance of statistical averaging by the ring arrays.

In reality, however, the situation is not as simple as it appears. The assumed ideal link of the ring to the leads is equivalent to the strong coupling limit in terms of a connectivity parameter.⁵ The implied absence of backscattering is at odds with the interpretation of the observed oscillations in terms of AAS oscillations due to coherent backscattering.^{4,6} Furthermore, the experimental samples^{3,4} were not single rings in the one-dimensional quantum limit, but a large array of connected rings, each containing several transport channels. The tuning of the Rashba spin-orbit parameter is associated with a strong variation in the electron density⁶ and therefore wave number of the interfering electrons. In the present Rapid Communication we offer an explanation

of the robustness of the observed AC oscillations with respect to the complications summarized above.

A quantitative analysis of the multi-mode ring array is very challenging and requires large scale numerical simulations.⁷ Here we proceed from a single single-mode quantum ring,² taking backscattering into account by assuming weak coupling to the electron leads. Its conductance can be understood as resonant tunneling through discrete eigenstates at the Fermi energy⁵ that are modulated by the SOI Rashba parameter. In-plane magnetic field⁸ allows tuning of the conductance oscillations without interference of the AB oscillations (see Fig. 1). We consider a modulation of the Rashba interaction strength that is associated with an experimentally known large change in the electron density.⁶ Small deviations between different rings in nanofabricated arrays can be taken into account by an ensemble averaging over slightly different single rings. We find that this procedure leads to an agreement with experiments that rivals previous theories. We consider a ring with a radius of R , defined

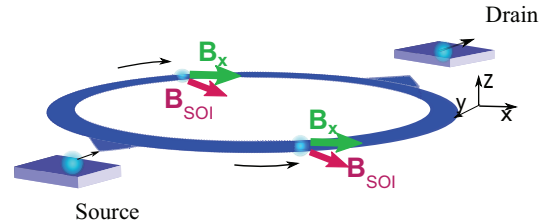


FIG. 1: Schematic of a quantum ring weakly coupled to source and drain contacts in the presence of SOI effective field, B_{SOI} , and in-plane magnetic field, B_x .

in the high-mobility two-dimensional electron gas in the x - y -plane. The Rashba SOI with the strength α is tunable by an external gate potential. The Hamiltonian of an electron in the ring has the form⁹

$$\hat{H}_{1D}^{(0)} = \frac{\hbar^2}{2mR^2} \left(-i \frac{\partial}{\partial \varphi} \right)^2 - \frac{\alpha}{R} (\cos \varphi \hat{\sigma}_x + \sin \varphi \hat{\sigma}_y) \left(i \frac{\partial}{\partial \varphi} \right) - i \frac{\alpha}{2R} (\cos \varphi \hat{\sigma}_y - \sin \varphi \hat{\sigma}_x), \quad (1)$$

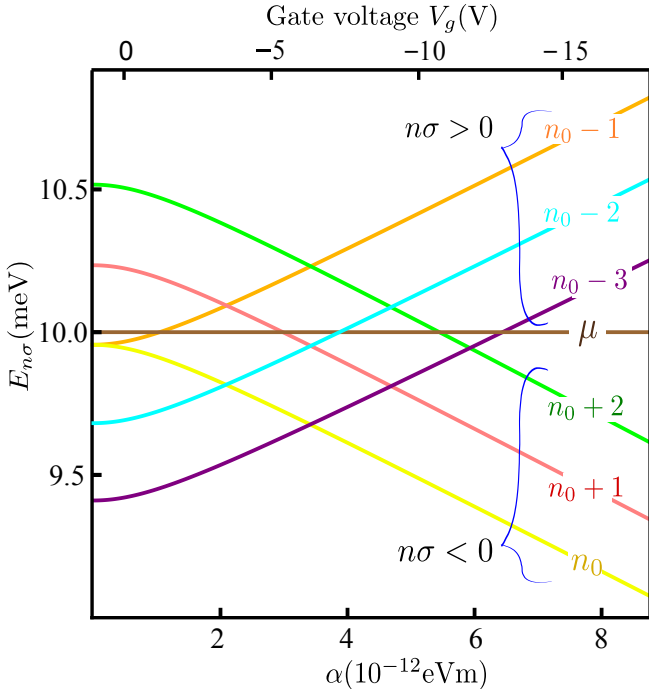


FIG. 2: Energies of a quantum ring with the radius $R = 630$ nm close to the Fermi energy $\mu = 10$ meV as function of the SOI strength α . Energies are labeled as $n_0 + i$ and $n_0 = 72$, for $n > 0$, whereby, each level is Kramers degenerate with $-n_0 - i - 1$ and opposite spin direction. The effective mass for InGaAs, $m = 0.045 m_0$, where m_0 is the electron mass. The conductance is nonzero when μ crosses an energy level.

where m is the effective mass, φ is the azimuthal angle, and $\hat{\sigma}_i$ are the Pauli matrices in the spin space. The eigenstates are

$$E_{n\sigma}^{(0)} = E_R \left[\left(n + \frac{1}{2} \right)^2 + \frac{1}{4} + \sigma \frac{n + \frac{1}{2}}{\cos \theta} \right], \quad (2)$$

where $E_R = \hbar^2 / (2mR^2)$, $\tan \theta = 2mR\alpha / \hbar^2$, the integer n is the angular momentum quantum number, and $\sigma = \pm$ denotes the spin degree of freedom.

In-plane magnetic field B along the x -direction contributes the Zeeman energy $H' = E_B \hat{\sigma}_x$, where $E_B = g\mu_B B / 2$, μ_B is the Bohr magneton, and g is the effective g -factor. We assume that the Zeeman energy is small compared to the (kinetic) Fermi energy and is treated as perturbation to the zero-field Hamiltonian, $H_{1D}^{(0)}$.

To leading order in E_B the energies $E_{n\sigma}^{(0)}$ are shifted by the in-plane field as:

$$\Delta_{n\sigma}^{(2)} = \frac{E_B^2}{8E_R} \left[\frac{\sin^2(2\theta)}{(2n \cos \theta + \sigma)(2(n+1) \cos \theta + \sigma)} + \frac{4 \sin^4 \frac{\theta}{2} \cos \theta}{n(\cos \theta + \sigma)} - \frac{4 \cos^4 \frac{\theta}{2} \cos \theta}{(n+1)(\cos \theta - \sigma)} \right]. \quad (3)$$

The gate voltage V_g modifies the asymmetry of the electron confinement potential, thereby modulating the

Rashba SOI strength α . We discuss here first the effects of varying SOI for constant Fermi energy and subsequently take the gate-induced density variation into account. In the absence of a magnetic field, the energy levels move with α according to Eq. (2). The four-fold degeneracy in the absence of SOI $E_{n,\sigma} = E_{n,-\sigma} = E_{-n-1,-\sigma} = E_{-n-1,\sigma}$ is broken when $\alpha \neq 0$ into two Kramers-degenerate doublets with $E_{n,\sigma} = E_{-n-1,-\sigma}$, see Eq. (2). For $\sigma n > (<) 0$ the energy increases (decreases) with α as indicated in Fig. 2.

Resonant tunneling occurs when the energy of the highest occupied level in the quantum ring, $E_{n_F,\sigma}$, equals the chemical potential μ in the leads, *i.e.* $E_{n_F,\sigma}(\alpha) = \mu$, as indicated in Fig. 2. Doublets of spin-split conductance peaks merge when $\alpha = 0$, $\mu = E_{n_F,\sigma}$, and the conductance becomes twice as large. The in-plane magnetic field shifts the energy levels as $\propto B^2$. As illustrated in Fig. 3, the resonant tunneling peaks at $E_{n_F,\sigma}(\alpha, B) = \mu$ are spin-split and non-parabolic. Fig. 3 agrees qualitatively with the experiments⁸ when assuming the strong coupling limit and justifying the apparent independence on the large μ variation with gate voltage by coherent backscattering. In the following we suggest an alternative interpretation.

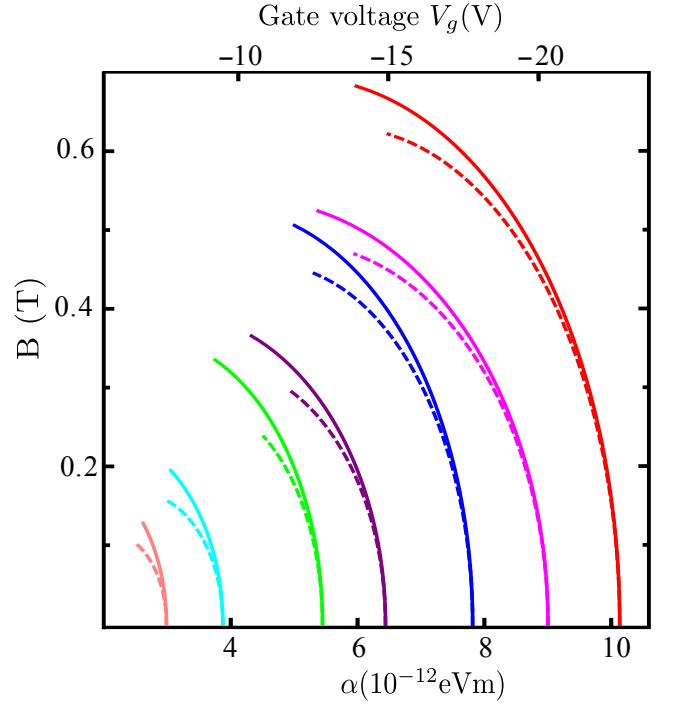


FIG. 3: Shift of the conductance peaks, that for zero magnetic field coincide with the crossings of the Fermi energy in Fig. 2, by an in-plane magnetic field as obtained by perturbation theory. The magnetic field is seen to break Kramers spin degeneracy. The parameters are the same as in Fig. 2 and $g = -2.9$ for InGaAs.

According to the experiments,^{3,4} α depends on the gate voltage as $\alpha [10^{-12} \text{ eVm}] = 0.424 - 0.47 \times V_G [10^{-12} \text{ V}]$ and on the electron density as $\alpha [10^{-12} \text{ eVm}] = 7.81 -$

$3.32 \times N_s [10^{12} \text{cm}^{-2}]$. In Fig. 4 we plot the ring energies as a function of α including the chemical potential μ that varies much faster with α than the single particle energies, leading to conductance peaks that as a function of gate voltage are very closely spaced. In ring arrays^{3,4} we do not expect to resolve such narrow resonances due to disorder, multi-mode contributions and ring size fluctuations. We can model the latter by averaging over an ensemble of rings with a Gaussian distribution of resonant energies or conductance peak positions with a phenomenological broadening parameter Γ . Fig. 5 illustrates the result of the averaging procedure in the form of the normalized conductance modulations.¹⁰ While the resonant tunneling peaks are smeared out, slow (AC) oscillation as a function of α reappears, which represents the beating of the level spacings induced by the SOI, in qualitative agreement with experiments.

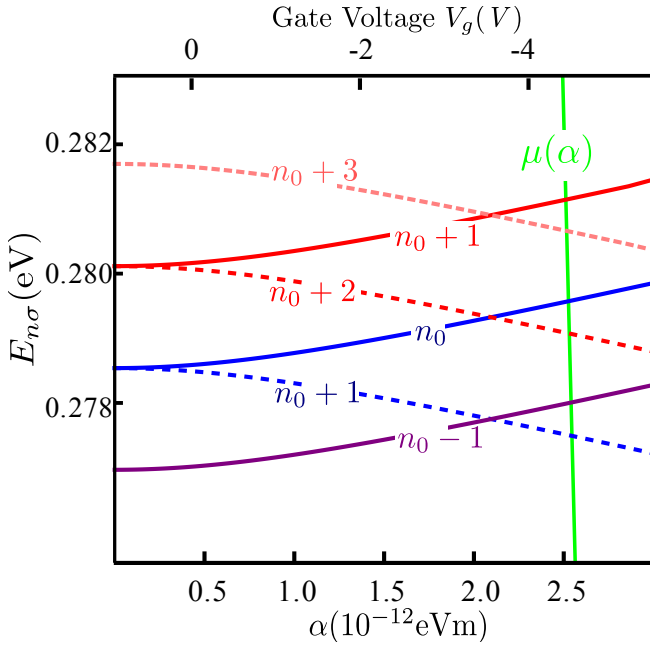


FIG. 4: Energy levels around the Fermi energy for $\alpha \approx 2.4 \times 10^{-12} \text{ eVm}$, compared to μ that strongly depends on the gate voltage that tunes α . Here, $n_0 = 380$, dashed and solid lines represent $n\sigma < 0$, and $n\sigma > 0$, respectively. Similar to Fig. 2, we labeled the energies for $n > 0$, keeping in mind that the energy levels are twofold degenerate.

The experiments of AC oscillations in arrays with different ring radii⁴ are compared in Fig. 6 with our results and that of a single open ring.

In Fig. 5, we also illustrate the effect of in-plane magnetic field on the ensemble of Rashba rings. The magnetic field shifts the phase of the oscillation to lower values of the gate voltage or larger α and thus suppresses the amplitude of the conduction oscillations increasingly for lower values of the gate voltage. These features agree again qualitatively with those observed experimentally by Nitta *et al.*⁸, although our theory appears to overestimate phase shifts at large negative gate volt-

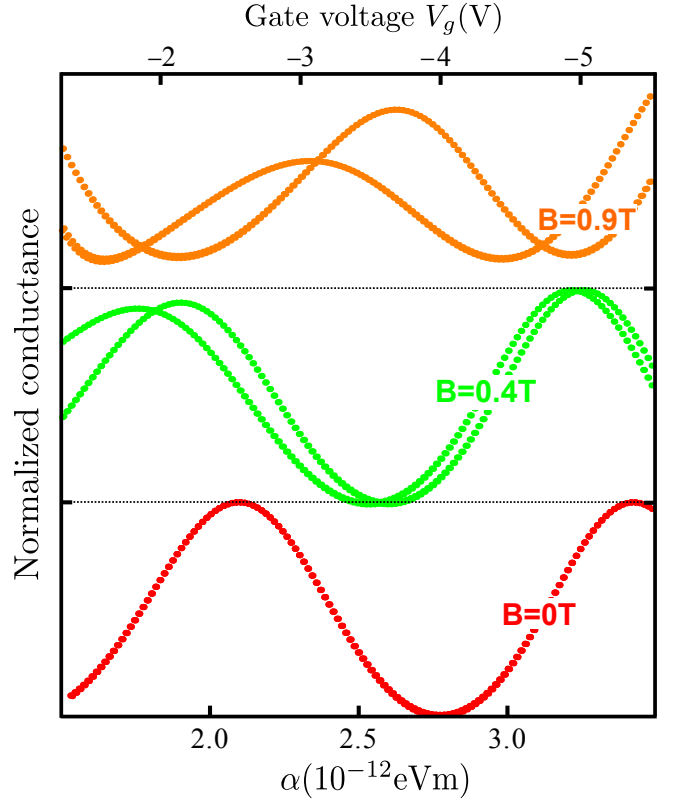


FIG. 5: The conductance oscillations of an ensemble of rings with energy levels broadened by a Gaussian with $\Gamma = 0.003 \text{ peV/m}$ as a function of an in-plane magnetic field. The dashed lines are guides to the eye, to compare the oscillation amplitudes while varying the magnetic field. All amplitudes are scaled with those at $B = 0 \text{ T}$ that display a modulation of $(G_{\max} - G_{\min})/G_{\max} = 50\%$. The in-plane magnetic field splits Kramers degenerate spin states that evolve differently with gate voltage.

ages. The magnetic field splits the Kramers degeneracy, thereby leading to two sets of superimposed oscillations that might be experimentally resolved in the form of different Fourier components.

Previous theories^{2,9} treat ideally open single rings, while we consider the weak coupling limit. Both extremes are likely not met in experiments. The intermediate regime can be modeled in terms of a connectivity parameter.⁵ An increased coupling causes a Lorentzian smearing of the conductance peaks, which is likely to effectively enhance the phenomenological broadening of the ensemble average and cannot be resolved in the experiments. The presence of several occupied modes in the rings also contributed to the average, since each radial node can be approximated as a ring with a slightly different radius. We therefore believe that our results are robust with respect to deviations from our Hamiltonian and these deviations can be captured by the phenomenological broadening parameter Γ .

In conclusion, we investigated the conductance of single rings and an ensemble of them as a function of the

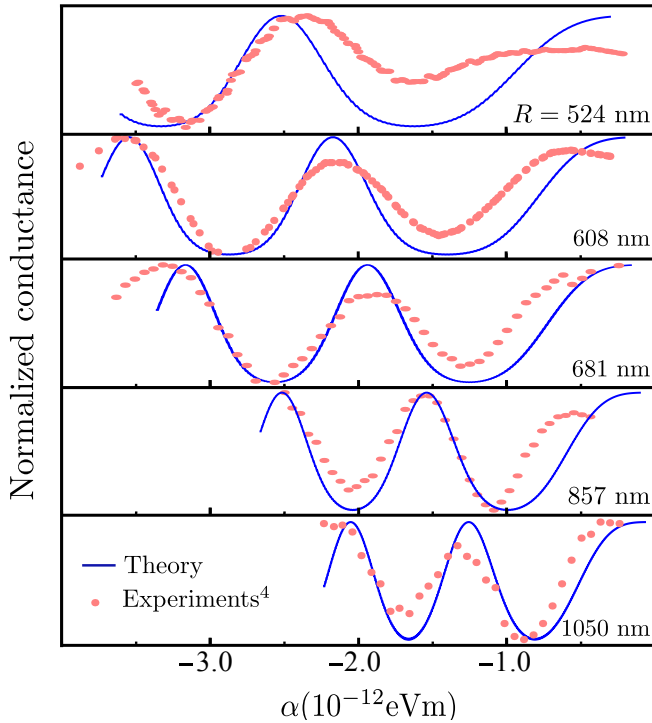


FIG. 6: The conductance G of a array of rings modeled as an ensemble energy levels as a function of α broadened by a Gaussian for various nominal radii R . The broadening parameters are $\Gamma = 0.005, 0.0035, 0.003, 0.002$ and 0.001 peV/m, for $R = 524; 608; 681; 857$ and 1050 nm, respectively. All amplitudes are scaled to a panel height corresponding to $(G_{max} - G_{min})/G_{max} = 50\%$. We use the experimentally determined relations between Rashba constant and electron density as before. We compare our calculations (lines) with the experimental results (points) from Ref. 4 (see also Ref. 10).

Rashba spin-orbit interaction in the limit of weak coupling to the leads. We considered both constant and gate voltage-dependent density of electrons. Both situations can in principle be realized experimentally by two independent (top and bottom) gate voltages. We compare results with experiments on ring arrays in which a single gate changes both the SOI α as well as the electron density. We found that, in agreement with experiments, the ensemble averaged conductance oscillates as a function of α . The oscillations undergo a phase shift under an in-plane magnetic field, and the period varies with the ring diameter, as observed. We conclude that experiments observe SOI-induced interference effects that are more complicated than the original Aharonov-Casher model but are robust with respect to the model assumptions.

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¹⁰ Experiments report resistances rather than conductances. The resistance oscillations $\delta R \ll R$ are measured on top of a large background resistance R . Thus, experiments are represented by $\delta R(\alpha \neq 0)/\delta R(\alpha = 0) = \delta G(\alpha \neq 0)/\delta G(\alpha = 0)$, which we may compare directly with our calculated normalized conductance oscillations.